COMP 2190 – Scientific Computing

Calendar Description: An applied computational course introducing topics such as approximation by polynomials, solution of non-linear equations, linear systems, simulation and computational geometry.
Prerequisites: COMP 1020 and one of MATH 1500, MATH 1501, MATH 1510, MATH 1520, MATH 1530 or MATH 1690.
Corequisite: One of MATH 1300, MATH 1301or 1310.
This course is a prerequisite for: COMP 3140 and COMP 3490.

Outline

1) Round-off Error, Floating Point Numbers (1/2 week)
   Binary floating point representation, accumulation of round-off error, programs to illustrate round-off error.
2) Polynomial Approximations (1 week)
   Lagrange polynomials, linear and quadratic approximation, the idea of a piece-wise approximation (spline), cubic approximation, problems with polynomials of high degree. Graphing functions using Maple, Mathematica, or Matlab.
3) Taylor Series, Power Series (1 week)
   Summing a geometric series, Taylor series, Maclaurin series, approximation using the first few terms, general power series, convergence, Cauchy ratio test.
4) Root Finding (1 ½ weeks)
   Bisection method, secant method, Newton's method, quadratic convergence, doubling of the number of correct digits, oscillations, single roots, double roots, rate of convergence with double roots, polynomial long division.
5) Least Squares (1/2 week)
   Fitting a straight line to a set of data points, summing the squares of errors, finding the minimum, transforming various curves into straight lines using substitutions of variables.
6) Iterated Functions, Dynamical Systems (1 ½ weeks)
   Convergence of an iterated function $x_{n+1} = g(x_n)$, i.e., a dynamical system. Fixed points of an iteration, attracting points, repelling points, indifferent points. Cycles of an iterated system -- 2-cycles, 3-cycles, 4-cycles, etc. Bifurcation. Graphing regions of attraction.
7) Random numbers (1 week)
   Generation of random numbers by computer, the "linear congruential method". Monte-Carlo simulation (dice, cards, etc). Estimating probabilities by simulation.
8) Parametric Curves, Vectors (1 week)
   Vectors [x,y] in the plane, addition of vectors, cross-product, dot-product, area, vectors [x(t),y(t)] which are functions of a parameter t, parametric curves, tangent vectors, Bezier curves.
9) The Predictor-Corrector method (1 week)
   Solving simple equations $dx/dt = f(x,t)$ numerically. The Euler predictor, the corrector. Solving two-variable systems, converting $d^2x/dt^2 = f(x,t)$ to a simple two variable system.
10) Introduction to Computational Geometry (1 week)
   The line segment connecting two points. The interior of a triangle determined by three points, convex regions in the plane, convex polygons. A line divides the plane into half-planes; which side of a line a point is on; testing whether a polygon is convex; testing whether a point P is inside a convex polygon.
11) Predator-Prey Models, Markov Chains (1 week)
   Simple linear predator-prey models, i.e., 2D linear dynamical systems; the use of probabilities gives Markov chains.
12) Area under a Curve (1 week)
   Estimating the area under a curve using the trapezoidal rule, and Simpson's rule. Derivations using line segments, parabolas, coding these.
13) Gaussian Elimination (1 week)
   Solving linear equations efficiently, representation as a matrix, round-off error, pivoting.

Text: Peter R. Turner, *Guide to Scientific Computing*, CRC Press, 2001